**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: Option B

Given Mean = 45 minutes, Standard Deviation = 8 mins.

The service starts after 10 mins of drop-off and completes in 1 hour within an hour.

The work can be completed within 50 mins i.e., 60 – 10 = 50 mins (1 hour = 60 mins).

So the service manager cannot meet his commitment when the time exceeds 50 mins.

probability that the service manager cannot meet his commitment =

(1- pnorm(50,45,8)) = 0.267

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: False. First calculate Z – score for 38 and 44

Z44 = (44-mean)/sd = (44-38)/6 = 1 (one standard deviation above the mean).

Z38 = (38-mean)/sd = (38-38)/6 = 0 (equals to mean)

So, the statement is false. Because the same number of employees falling within the ranges of 38-44 and older than 44.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True. Calculate Z – Score for age 30.

Z = (30-38)/6 = -1.33

P(X ≤ 30) = p(Z ≤ -1.33) = 0.0918 (by z table)

So, the employees attracted to the training program can be = 9.1% of 400 = 400\*9.1%=36.4 (approx.) = 36.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: Both the Distributions are normally distributed but the parameters varies, the explanation will be given below.

**Distribution of 2 *X*1:** In this distribution, When we are multiplying with the constant (2) the mean is multiplied with 2 but the variance is multiplied with the square of the constant i.e., ‘4’.

2*X1* ~ *N*(2μ, 4σ2)

**Distribution of *X*1 + *X*2**: In this we are adding the individual variables so the mean is doubled and same with the variance. *X1 + X2* ~ *N*(2μ,2σ2)

**Parameters of 2 *X*1**: μ, σ2 these are the initial values

Mean = 2 μ (doubled mean)

Variance = 4σ2 (4times of the original variance)

**Parameters of 2 *X*1**: μ, σ2 these are the initial values

Mean: 2 μ = (μ+ μ)

Variance = 2 σ2 (σ2 + σ2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

Ans: Option D

We have to find out quantile normal distribution i.e.,

qnorm(0.995, 100, 20) = 151.5

qnorm(0.005, 100, 20) = 48.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: We have to find out the Quantile Distribution for the 2 profit Distributions.

For first profit distribution:

qnorm(0.975,225,3) = 230.8799

qnorm(0.025,225,3) = 219.1201

Range (Million Rupee) = [219.12,230.88]

For second distribution:

qnorm(0.975,315,4) = 322.8399

qnorm(0.025,315,4) = 307.1201

Range (Million Rupee ) = [ 307.16, 322.84]

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans : 5th quantile distribution for profit distribution

qnorm(0.05,225,3) = 220.0654

qnorm(0.05,315,4) = 308.4206

Total 5th percentile distribution = 220.0654+308.4206 = 528.486 (million rupee)

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: The easy way to determine which the larger probability of making loss is the mean which is nearer to zero or negative value. By comparing the means of two distribution we can say that Division1(5 $million) has more probability of making loss than the Divison2(7 $million) in the given year.